

AutF2

Computations for the automorphisms group of F2

1.0

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Chapter 1

Automorphisms

1.1 Automorphisms

1.1.1 IsAutomorphismOfF2 (for IsObject)

▷ IsAutomorphismOfF2(*arg*) (filter)
Returns: true or false

1.1.2 AutomorphismOfF2 (for IsFreeGroup, IsList)

▷ AutomorphismOfF2(*F*, *list*) (operation)

Constructor of the automorphism object given a word of automorphisms.

1.1.3 WordOfAutomorphismOfF2 (for IsAutomorphismOfF2)

▷ WordOfAutomorphismOfF2(*aut*) (attribute)

Returns the word of the given automorphism.

1.1.4 ImagesAutomorphismOfF2 (for IsAutomorphismOfF2)

▷ ImagesAutomorphismOfF2(*aut*) (attribute)

Returns the images of the generators of F_2 by the given automorphism.

1.1.5 ImageByAutomorphismOfF2 (for IsAutomorphismOfF2, IsAssocWordWithInverse)

▷ ImageByAutomorphismOfF2(*aut*, *w*) (operation)

Returns the image of the given word by the given automorphism.

1.1.6 IsIdentityAutomorphismOfF2 (for IsAutomorphismOfF2)

- ▷ `IsIdentityAutomorphismOfF2(aut)` (property)
Returns: true or false

1.1.7 MatrixRepresentationOfAutomorphismOfF2 (for IsAutomorphismOfF2)

- ▷ `MatrixRepresentationOfAutomorphismOfF2(aut)` (attribute)
 Returns the representation of an automorphism in $GL_2(\mathbb{Z})$.

1.1.8 IsSpecialAutomorphismOfF2 (for IsAutomorphismOfF2)

- ▷ `IsSpecialAutomorphismOfF2(aut)` (attribute)
 Returns whether or not an automorphism is special, that is its representation in $GL_2(\mathbb{Z})$ has determinant 1.

1.1.9 IsConjugacyAutomorphismOfF2 (for IsAutomorphismOfF2)

- ▷ `IsConjugacyAutomorphismOfF2(aut)` (property)
Returns: true or false

1.1.10 ConjugacyElementConjugacyAutomorphismOfF2 (for IsAutomorphismOfF2)

- ▷ `ConjugacyElementConjugacyAutomorphismOfF2(aut)` (attribute)
 Getter of the element that determines a conjugacy automorphism

1.1.11 ConjugacyAutomorphismOfF2 (for IsFreeGroup, IsAssocWordWithInverse)

- ▷ `ConjugacyAutomorphismOfF2(F, w)` (operation)
 Returns the automorphism determined by the conjugacy of the given word in F_2

1.2 Recognizing automorphisms

1.2.1 AutomorphismOfF2ByImages (for IsFreeGroup, IsAssocWordWithInverse, IsAssocWordWithInverse)

- ▷ `AutomorphismOfF2ByImages(F, u, v)` (operation)
 Returns an automorphism such that the images of the basis are the given words *u* and *v*. If the automorphism does not exist, returns false.

1.2.2 AutomorphismOfF2ByMatrix (for IsFreeGroup, IsList)

▷ AutomorphismOfF2ByMatrix(F , M)

(operation)

Returns an automorphism that has the given matrix as image in $GL_2(\mathbb{Z})$.

Chapter 2

Conjugacy, fixed points subgroup and centralizer

2.1 Centralizer

2.1.1 CentralizerAutomorphismOfF2

▷ `CentralizerAutomorphismOfF2(aut)` (function)

Returns a generating set of the centralizer of *aut*.

2.2 Conjugacy

2.2.1 AreConjugateAutomorphismsOfF2

▷ `AreConjugateAutomorphismsOfF2(a, b)` (function)

Returns a conjugating element if the automorphisms *a* and *b* are conjugated. Otherwise the function returns false.

2.3 Fixed points subgroup

2.3.1 FixedSubgroupAutomorphismOfF2

▷ `FixedSubgroupAutomorphismOfF2(aut)` (function)

Returns a basis of the fixed point subgroup of *aut*.

Chapter 3

Related Algorithms

3.1 Matrix groups

3.1.1 ConjugacyGL2Z

▷ `ConjugacyGL2Z(A, B)` (function)

Decides whether a pair of 2x2 matrices in $GL_2(\mathbb{Z})$ are conjugate. If so returns the element C such that $A = C^{-1}BC$.

3.1.2 CentralizerGL2Z

▷ `CentralizerGL2Z(A, B)` (function)

Given a matrix A , computes a matrix C such that $C_{GL_2(\mathbb{Z})}(A) = \langle -C, C \rangle$ and an integer n such that $C^n = A$.

3.1.3 MembershipSubgroupSL2Z

▷ `MembershipSubgroupSL2Z(gens, A)` (function)

Given a matrix A , decides whether $A \in \langle gens \rangle$ and if so computes a word w in the generators that represent A .

3.2 Free groups

3.2.1 AreAutomorphicEquivalent (for IsFreeGroup, IsAssocWordWithInverse, IsAssocWordWithInverse)

▷ `AreAutomorphicEquivalent(F, u, v)` (operation)

Decides whether u and v are automorphic equivalent in F .

3.2.2 IsCyclicallyReducedWord (for IsAssocWordWithInverse)

- ▷ IsCyclicallyReducedWord(u) (property)
Returns: true or false

3.2.3 IsSubword (for IsAssocWordWithInverse, IsAssocWordWithInverse)

- ▷ IsSubword(u, v) (operation)

Returns whether the word v is a subword of u or not.

3.2.4 IsPrefix (for IsAssocWordWithInverse, IsAssocWordWithInverse)

- ▷ IsPrefix(u, v) (operation)

Returns whether the word v is a prefix of u or not.

3.2.5 IsSuffix (for IsAssocWordWithInverse, IsAssocWordWithInverse)

- ▷ IsSuffix(u, v) (operation)

Returns whether the word v is a suffix of u or not.

3.2.6 RootFreeGroup (for IsAssocWordWithInverse)

- ▷ RootFreeGroup(v) (operation)

Returns a word w and an integer r such that $w^r = v$ and r is maximal.

3.2.7 NielsenReducedSet

- ▷ NielsenReducedSet(V) (function)

Given a sequence of elements V of a free group, returns a Nielsen reduced set W that is equivalent to V .

3.2.8 IsNielsenReducedSet

- ▷ IsNielsenReducedSet(V) (function)

Given a sequence of elements V of a free group, returns whether V is a Nielsen reduced set or not.

3.2.9 InFreeSubgroup

- ▷ InFreeSubgroup(V, w) (function)

Given a sequence of elements V of a free group and a word w , returns whether w is in the subgroup generated by V or not.

3.2.10 IsSubgroupOfFreeSubgroup

▷ `IsSubgroupOfFreeSubgroup(V, W)` (function)

Given two sequence of elements V, W of a free group and a word w , returns whether the subgroup generated by W is a subgroup of the group generated by V or not.

3.2.11 IsNormalSubgroupOfFreeSubgroup

▷ `IsNormalSubgroupOfFreeSubgroup(V, W)` (function)

Given two sequence of elements V, W of a free group and a word w , returns whether the subgroup generated by W is a normal subgroup of the group generated by V or not.

3.2.12 AreEquivalent

▷ `AreEquivalent(V, W)` (function)

Given two sequence of elements V, W of a free group and a word w , returns whether V and W are equivalent or not.

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